



Cambridge International AS & A Level

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MATHEMATICS

9709/33

Paper 3 Pure Mathematics 3

October/November 2020

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Blank pages are indicated.

- 2 On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z| \geq 2$ and $|z - 1 + i| \leq 1$. [4]

3 The parametric equations of a curve are

$$x = 3 - \cos 2\theta, \quad y = 2\theta + \sin 2\theta,$$

for $0 < \theta < \frac{1}{2}\pi$.

Show that $\frac{dy}{dx} = \cot \theta$.

[5]

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4 Solve the equation

$$\log_{10}(2x + 1) = 2 \log_{10}(x + 1) - 1.$$

Give your answers correct to 3 decimal places.

[6]

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- 5 (a) By sketching a suitable pair of graphs, show that the equation $\operatorname{cosec} x = 1 + e^{-\frac{1}{2}x}$ has exactly two roots in the interval $0 < x < \pi$. [2]

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- (b) The sequence of values given by the iterative formula

$$x_{n+1} = \pi - \sin^{-1} \left(\frac{1}{e^{-\frac{1}{2}x_n} + 1} \right),$$

with initial value $x_1 = 2$, converges to one of these roots.

Use the formula to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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6 (a) Express $\sqrt{6} \cos \theta + 3 \sin \theta$ in the form $R \cos(\theta - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. State the exact value of R and give α correct to 2 decimal places. [3]

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- (b) Hence solve the equation $\sqrt{6} \cos \frac{1}{3}x + 3 \sin \frac{1}{3}x = 2.5$, for $0^\circ < x < 360^\circ$. [4]

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7 (a) Verify that $-1 + \sqrt{5}i$ is a root of the equation $2x^3 + x^2 + 6x - 18 = 0$. [3]

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(b) Find the other roots of this equation.

[4]

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8 The coordinates (x, y) of a general point of a curve satisfy the differential equation

$$x \frac{dy}{dx} = (1 - 2x^2)y,$$

for $x > 0$. It is given that $y = 1$ when $x = 1$.

Solve the differential equation, obtaining an expression for y in terms of x . [6]

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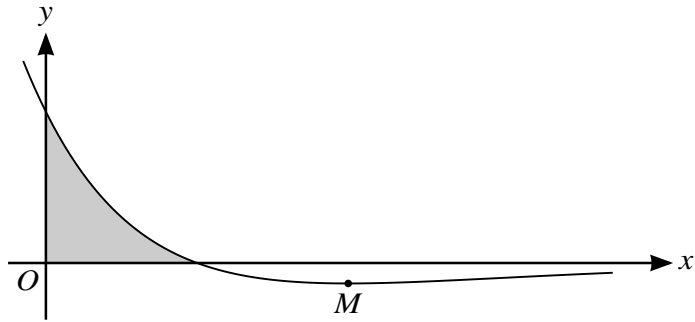
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A series of 20 horizontal dotted lines for writing.

10



The diagram shows the curve $y = (2 - x)e^{-\frac{1}{2}x}$, and its minimum point M .

- (a) Find the exact coordinates of M . [5]

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(b) Find the area of the shaded region bounded by the curve and the axes. Give your answer in terms of e . [5]

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11 Two lines have equations $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda(a\mathbf{i} + 2\mathbf{j} - \mathbf{k})$ and $\mathbf{r} = 2\mathbf{i} + \mathbf{j} - \mathbf{k} + \mu(2\mathbf{i} - \mathbf{j} + \mathbf{k})$, where a is a constant.

(a) Given that the two lines intersect, find the value of a and the position vector of the point of intersection. [5]

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- (b) Given instead that the acute angle between the directions of the two lines is $\cos^{-1}\left(\frac{1}{6}\right)$, find the two possible values of a . [6]

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Additional Page

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